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Definition of Probability Based on Already Happened Outcomes: Application in Identifying Rainy and Non-Rainy Period

Dhritikesh Chakrabarty

Independent Researcher, (Ex Associate Professor of Statistics, Handique Girls' College, Guwahati, Assam, India)

Abstract - The concept of empirical probability has been applied in defining the probability of occurrence of rainfall in terms of rainy days on the basis of the data on already happened outcomes and then the definition has been applied in estimating probabilities of occurrences of different numbers of rainy days at four stations in India namely Chennai, Kolkata, Mumbai and New Delhi with an objective of identifying rainy & non-rainy periods at these stations. It has been found from the study that at each of the four stations there does not exist any month which is certain to be completely non-rainy while at Mumbai, the period January – April is almost certain to be non-rainy and that the periods September – November, June – September, July – September & July – August are certain to be rainy at Chennai, Kolkata, Mumbai & New Delhi respectively while the period June – August is the common rainy period of these four stations.

Keywords: Probability, rainy days, rainy period, non-rainy period, identification.

1. INTRODUCTION

Probability is a basic statistical tool of understanding and explaining of various phenomena in almost every branch of science [2 , 7 , 28 , 29 , 37 , 38]. The history of development of probability can be broadly classified into five broad eras namely.

(1) The Era of Prehistory of Probability Theory, whose beginning were lost in the dust of antiquity [7 , 38] ;

(2) The Era of Origin of Probability Theory as a Science, continued from the middle of the seventeenth century up to the beginning of the eighteenth century [2 , 38];

(3) Bernoullian Era of Probability Theory, begun with the appearance of the treatise "Arts Conjectandi", by James Bernoull [4] in 1713 and continued up to the early period of nineteenth century [7 , 38] ;

(4) The Era of Probability Theory in Russian School, continued from the middle of the nineteenth century up to the first quarter of the 20th century mainly with the works of Chebyshev, Markov and Liapounov [7 , 38]

and (5) The Era of Modern Probability Theory, begun with the formulations of axioms mainly by Bernstein [5 , 6] and Kolmogorov [7 , 31 , 32 , 38].

The theory of probability has come to the current stage through five approaches namely

(1) Subjective Approach introduced by Bayes [3 , 7 , 27];

(2) Intuitive Approach due to Koopman & Savage [27 , 41 , 42 , 43];

(3) Classical Approach due to Bernoull [4 , 9 , 12 , 13 , 27];

(4) Empirical Approach due to von Mises & Fisher which is also termed as relative frequency approach or statistical approach [11 , 25, 27 , 45 , 46 ,47 , 48 , 49 , 50];

(5) Axiomatic Approach due to Bernstein & Kolmogorov [5 , 6 , 7 , 27 , 28 , 31 , 32 , 38]

and (6) Theoretical Approach introduced by Chakrabarty during the first decade of the 21st century [8 , 10 , 12 , 13 , 14 , 15 , 16 , 17].

In every approach [29], as mentioned above, probability is defined or determined on the basis of random experiment either performing the actual experimentation or prior to performing it. In reality however, we observe experiment/phenomena which are not to be performed but happened automatically and which consist of several possible outcomes. In this case, we don't have scope of performing experimentation. What we can do; we can simply collect data from the already happened outcomes of the experiment. Probability of occurrence of event may be required to be calculated in such cases. In this paper, probability of occurrence of an event has been defined in such cases with numerical application.

There had already been lot of studies on rainfall on various aspects like measures of characteristics of rainfall [18 , 20 , 21 , 22 , 23 , 24, 26], trend of rainfall [1 , 19 , 30 , 35 , 36 , 39 , 40 , 44], forecasting on rainfall [18 , 20] etc. The studies on rainfall done so far are mostly based on non-probabilistic approach. Study on rainfall has however hardly been done so far by probabilistic approach. In the current study, the concept of empirical probability has been applied in defining the probability of occurrence of rainfall in terms of rainy days on the basis of the data on already happened outcomes and then the definition has been applied in estimating probabilities of occurrences of different numbers of rainy days at four stations in India namely Chennai, Kolkata, Mumbai and New Delhi with an objective of identifying rainy & non-rainy periods at these stations.

2. ALREADY HAPPENED OUTCOMES AND DEFINITION OF PROBABILITY

Probability is statistically defined as follows [25 , 29 , 45 – 50] :

Definition (1): If a trial is repeated N times under identical condition and if out of the N repetitions an event E occurs n times then the probability of occurrence of the event E, denoted by P(E), is a number towards which the ratio $\frac{n}{N}$ approaches as N becomes larger i.e.

$$\frac{n}{N} \rightarrow P(E) \quad \text{as} \quad N \rightarrow \infty$$

i.e. P(E) is the limiting value of $\frac{n}{N}$ as $N \rightarrow \infty$

Definition (2): If a trial is repeated N times under identical condition and if out of the N repetitions an event E occurs n times then the probability of occurrence of the event E, denoted by P(E), is a number such that the number of occurrence of the event E approaches N.P(E) as N becomes larger i.e.

$$n \rightarrow N.P(E) \quad \text{as} \quad N \rightarrow \infty$$

This definition is just the inverse versions of **Definition (1).**

This definition states that the number of occurrence of the event E out of N repetitions of the trial can be approximated by N.P(E) provided N is large.

The following fundamental properties of probability can be obtained from the definition:

Property (1): For any event E, $0 < P(E) < 1$.

P(E) = 0 iff E is impossible event P(E) = 1 iff E is certain event.

Property (2): Probability of non-occurrence of the event is 1 – P(E) .

Property (3): If a trial results in N possible outcomes e_1 , e_2 , ………… , e_N then the probability of occurrence of either any of the N outcomes is 1 .

If the N possible outcomes are equally likely then the probability of occurrence of each of the N outcomes is $\frac{1}{N}$

i.e. $P(e_1) = P(e_2) = \ldots\ldots\ldots = P(e_N) = \frac{1}{N}$

If the N possible outcomes are mutually exclusive then the probability of occurrence of either any of the N outcomes is the sum of their individual probabilities of occurrences

i.e. $P(e_1 \cup e_2 \cup \ldots\ldots\ldots \cup e_N) = P(e_1) + P(e_2) + \ldots\ldots\ldots + P(e_N)$

Property (4): If A and B are mutually exclusive then the probability of occurrence of either A and B is the sum of the individual probabilities of occurrences of A and B

i.e. $P(A \cup B) = P(A) + P(B)$

In general, if E_1 , E_2 , ………… , E_n are mutually exclusive events then the probability of occurrence of either any of the n events is the sum of their individual probabilities of occurrences

i.e. $P(E_1 \cup E_2 \cup \ldots\ldots \cup E_n) = P(E_1) + P(E_2) + \ldots\ldots + P(E_n)$

Now suppose that E is a possible event associated to a natural phenomenon and that it has already happened N(E) times out of N repetitions of the phenomenon.

Since the phenomenon has happened naturally, it is free from error that occurs due to performing of experiment.

Moreover, the natural happening of the phenomenon can be thought of as the performing of experiment not by human but by nature.

Hence by Definition (1), P(E) the probability of occurrence of the event E is the limiting value of $\frac{N(E)}{N}$ as $N \to \infty$.

But for finite N (i.e. for finite sample size), the ratio $\frac{N(E)}{N}$ may not be the accurate value of P(E) . However, this ratio can be regarded as estimated value of P(E).

Automatically, the following results can be obtained from the properties of probability as mentioned above:

Result (1):

Estimate of probability of non-occurrence of the event E

= 1 − Estimate of probability of occurrence of E = 1 − Estimate P(E)

Result (2): If A and B are mutually exclusive then estimate of probability of occurrence of either A and is the sum of the estimates of individual probabilities of occurrences of A and B.

Result (3): In general, if E_1 , E_2 , ………… , E_n are mutually exclusive events then estimate of probability of occurrence of either any of the n events is the sum of their estimated individual probabilities of occurrences.

3. APPLICATION OF THE DEFINITION TO NUMERICAL DATA

The definition of probability based on the data on already happened outcomes has been applied in estimating probability of occurrence of number of rainy days in different months at the four stations in India namely Chennai, Kolkata, Mumbai and New Delhi. For this purpose, data on number of rainy days (month-wise) at these four stations have been collected from the year 1969 onwards from Meteorological Department of Government of India and then the above formulation of probability has been applied in computing the desired values of probabilities.

The following probabilities have been considered for estimation for each of the 12 months at each of the 4 stations:

(i) Probability of occurrence of 0 rainy day (i.e. completely non-rainy day).

(ii) Probability of occurrence of exactly 1 rainy day.

(iii) Probability of occurrence of either 0 rainy day or 1 rainy day.

(iv) Probability of occurrence of at least 1 rainy day (i.e. 1 or more rainy days).

(v) Probability of occurrence of at least 2 rainy day (i.e. 2 or more rainy days).

The estimated values obtained have been presented in **Table – 5.1, Table – 5.2, Table – 5.3,** and **Table – 5.4** for the stations Chennai, Kolkata, Mumbai and New Delhi respectively.

4. RESULT AND DISCUSSION

By the **Property (1)** of probability, if the probability of occurrence of zero rainy day at a place during a month/period is 1 then the month/period can be regarded as a certain to be non-rainy one.

In reality, there may be rainfall during a non-rainy period due to some random cause that occurs accidently but not regularly and not always so that 1 rainy day can occur during a non-rainy month with very small (near to 0) probability. Thus, if the probability of occurrence of zero rainy day during a period is not 1 but near to 1 and the probability of occurrence of 1 rainy day during the period is very small such that the probability of occurrence of either 0 rainy day or 1 rainy day is 1 (i.e. there are only 2 possible outcomes namely 0 and 1) then the period can be regarded as almost certain to be non-rainy period.

Similarly, if the probability of occurrence of 2 or more rainy days (i.e. at least 2 rainy days) is 1 then the period can be regarded as a certain rainy one.

Similarly, if the probability of occurrence of 2 or more rainy days is very near to 1 and the probability of occurrence of at least 1 rainy day is 1 then the period can be regarded as an almost certain rainy one.

In similar manner, the period can be regarded as more likely to be rainy period, equally likely to be rainy period or less likely to be rainy period depending upon the probability of occurrence of rainy days.

From the estimated values of the probabilities as shown in the tables (Table Numbers from 5.1 to 5.4), the estimated rainy and non-rainy months/periods at the four stations have been obtained as shown in **Table – 5.5.**

Some special information obtained from the findings of this study are as follows:

(1) There does not exist any month which is certain to be completely non-rainy. However, at Mumbai, the period January – April is almost certain to be completely non-rainy.

(2) The periods September – November, June – September, July – September & July – August are certain to be rainy at Chennai, Kolkata, Mumbai & New Delhi respectively.

(3) The period June – August is the common rainy period of these four stations.

5. TABLES OF FINDINGS

Table -5.1: (Estimated probability of occurrence of rainy day at Chennai)

Month	Estimated value of probability of occurrence of				
	0 rainy day	**Exactly 1 rainy day**	**Either 0 or 1 rainy day**	**At least 1 rainy day**	**At least 2 rainy days**
January	0.5	0.1333	0.6333	0.5	0.3333
February	0.8	0.0667	0.8667	0.2	0.1333
March	0.7667	0.1667	0.9334	0.2333	0.0667
April	0.6	0.1333	0.7333	0.4	0.2667
May	0.3	0.3667	0.6667	0.7	0.3333
June	0	0.1	0.1	1	0.9
July	0	0.0667	0.0667	1	0.9333
August	0	0.0333	0.0333	1	0.9667
September	0	0	0	1	1
October	0	0	0	1	1
November	0	0	0	1	1
December	0.1	0.0667	0.1667	0.9	0.8333

Table -5.2: (Estimated probability of occurrence of rainy day at Kolkata)

Month	Estimated value of probability of occurrence of				
	0 rainy day	**Exactly 1 rainy day**	**Either 0 or 1 rainy day**	**At least 1 rainy day**	**At least 2 rainy days**
January	0.4286	0.25	0.6786	0.5714	0.2857
February	0.1429	0.3929	0.5358	0.8571	0.4643
March	0.3571	0.0714	0.4285	0.6429	0.5714
April	0.0833	0.2143	0.2976	0.9167	0.7143
May	0	0.0357	0.0357	1	0.9643
June	0	0	0	1	1

July	0	0	0	1	1
August	0	0	0	1	1
September	0	0	0	1	1
October	0.0357	0.0357	0.0714	0.9643	0.9286
November	0.3929	0.2143	0.6072	0.6071	0.3929
December	0.7143	0.0714	0.7857	0.2857	0.2143

Table -5.3: (Estimated probability of occurrence of rainy day at Mumbai)

Month	Estimated value of probability of occurrence of				
	0 rainy day	**Exactly 1 rainy day**	**Either 0 or 1 rainy day**	**At least 1 rainy day**	**At least 2 rainy days**
January	0.9524	0.0476	1	0.0476	0
February	0.9286	0.0714	1	0.0714	0
March	0.9762	0,0238	1	0.0238	0
April	0.9286	0.0714	1	0.0714	0
May	0.5714	0.0714	0.6428	0.4286	0.119
June	0.0238	0	0.0238	0.9762	0.9762
July	0	0	0	1	1
August	0	0	0	1	1
September	0	0	0	1	1
October	0.1429	0.0476	0.1905	0.8571	0.8095
November	0.4762	0.0476	0.5238	0.5238	0.4762
December	0.8333	0.0952	0.9285	0.2619	0.1667

Table -5.4: (Estimated probability of occurrence of rainy day at New Delhi)

Month	Estimated value of probability of occurrence of				
	0 rainy day	**Exactly 1 rainy day**	**Either 0 or 1 rainy day**	**At least 1 rainy day**	**At least 2 rainy days**
January	0.303	0.2121	0.5151	0.697	0.4848
February	0.1515	0.3333	0.4848	0.8485	0.4848
March	0.2727	0.2121	0.4848	0.7273	0.4545
April	0.3636	0.3636	0.7272	0.6364	0.2424

May	0.303	0.1212	0.4242	0.697	0.5455
June	0	0.0606	0.0606	1	0.9394
July	0	0	0	1	1
August	0	0	0	1	1
September	0.0606	0.0303	0.0909	0.9394	0.9697
October	0.4242	0.2424	0..6666	0.5758	0.303
November	0.7273	0.1212	0.8485	0.2727	0.1212
December	0.4848	0.2424	0.7272	0.5152	0.224

Table -5.5: (Estimated classification of period/month with respect to rainy day)

Month/Period	Station			
	Chennai	**Kolkata**	**Mumbai**	**New Delhi**
Certain to be non-rainy	Nil	Nil	Nil	Nil
Almost certain to be non-rainy	Nil	Nil	January – April	Nil
More likely to be non-rainy	February – April	December	May & December	November
Equally likely to be rainy & non-rainy	January	Nil	Nil	December
More likely to be rainy	May & December	January – April, October – November	October – November	January – May, September – October
Almost certain to be rainy	June – August	May	June	June
Certain to be rainy	September – November	June – September	July – September	July – August

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