Partners Universal International Innovation Journal (PUIIJ)
Volume: 03 Issue: 03 | May-June 2025 | ISSN: 2583-9675 | www.puiij.com

Combined Set of Several Sets of Observations: Geometric Mean

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Abstract - A formula has been derived for computing geometric mean of a set of observations which is a combined set of several sets of observations when the observations are unknown but the respective geometric means of the sets and the respective numbers of observations in the sets are known. Application of the formula in numerical data has also been shown along with the derivation of the formula.

Keywords: Observations, Several Sets, Combined Set, Geometric Mean, Formula.

1. INTRODUCTION

Geometric Mean [2 , 12] which is one of the three basic measures of average, introduced by Pythagoras, among whose the other two measures are arithmetic mean & harmonic mean [2 , 3 , 6] is a popular and important measure having wide applications in analysis of data. In statistics, it is used in measuring central tendency of data [13 , 14 , 16 , 17]. It seems that there are many situations where geometric meaning is suitable to be applied. It has been found that geometric mean is a reasonable measure of dispersion of data of ratio type [8]. It is also a reasonable tool for finding measure of relative change in a group of variables [5 , 7].

Lot of probable properties to be stratified by geometric mean are yet to be studied. Of course, its multiplicative property has been derived in a recent study [8 , 10]. On the other hand, its other probable applications are yet to be identified.

There are many situations where geometric means of several sets of observations are available but the observations are not available, and it is required to find out the geometric mean of all the unknown observations combined together. This type of situation often arises in meta-analysis in clinical trials [1 , 4 , 15]. Attempt has been made on deriving a formula for computing geometric mean of all the unknown observations in such situation. Application of the formula in numerical data has also been shown along with the derivation of the formula.

2. GEOMETRIC MEAN OF A SET OF NUMBERS

Definition

Let us consider a set of n real numbers or values namely

$$a_1 , a_2 , \ldots\ldots , a_n$$

Then the **Geometric Mean** of them is given by

$$G(\ldots) = G(a_1 , a_2 , \ldots\ldots , a_n) = (\prod_{i=1}^{n} a_i)^{1/n} \qquad (1)$$

provided the n numbers are strictly positive. [2 , 12].

Note (1):

The definition of geometric mean implies that

$$\prod_{i=1}^{n} a_i = \{\boldsymbol{G}(a_1, a_2, \ldots\ldots\ldots, a_n)\}^n \qquad (2)$$

Note (2):

As per the definition of geometric mean from first principle [6], geometric mean of the numbers

$$a_1, a_2, \ldots\ldots, a_n$$

is a number G(.....) which satisfies the equality

$$\prod_{i=1}^{n} a_i = \prod_{i=1}^{n} \boldsymbol{G}(\ldots..) \qquad (3)$$

This also implies the equation (2).

3. GEOMETRIC MEAN OF COMBINED SET

Suppose, there are k sets namely

$$S_1, S_2, \ldots\ldots, S_k$$

containing

$$n_1, n_2, \ldots\ldots, n_k$$

observations respectively such that geometric means of the respective sets are

$$g_1, g_2, \ldots\ldots, g_k$$

respectively.

The combined set S, of all these k sets contains

$$n_1 + n_2 + \ldots\ldots + n_k$$

observations.

By equation (2),

$$\prod_{j=1}^{n_i} x_j, (x_j \in S_i) = g_i^{n_i}, \ (i = 1, 2, \ldots\ldots, k)$$

This implies,

$$\prod_{i=1}^{k} \prod_{j=1}^{n_i} x_{ij}, (x_{ij} \in S) = \prod_{i=1}^{k} g_i^{n_i}$$

Therefore by equation (1), geometric mean g of the observations in the set S will be

$$g = \{g_1^{n_1} \times g_2^{n_2} \times \ldots\ldots\ldots \times g_k^{n_k}\}^{1/(n_1 + n_2 + \ldots\ldots + n_k)} \qquad (4)$$

This is the formula for computing geometric mean of a combined set of observations from a number of sets of observations when the observations are unknown but the geometric means of the set and the numbers of observations in the respective sets are known.

This formula has been stated in the form of a theorem below:

Theorem:

If $g_1, g_2, \dots\dots, g_k$ are geometric means of respective sets having $n_1, n_2, \dots\dots, n_k$ observations respectively then the geometric mean g of the set of all observations in the k sets combined together is given by

$$g = \{g_1^{n_1} \times g_2^{n_2} \times \dots\dots \times g_k^{n_k}\}^{1/(n_1 + n_2 + \dots\dots + n_k)}$$

In particular, if g_x & g_y are geometric means of two respective sets of observations then the geometric mean the geometric mean of the combined observations of the combined observations will be

$$g_{xy} = \{ g_x^{m} \times g_y^{n} \}^{1/(m+n)}$$

Note (3):

The formula given by equation (2.4), also stated in the theorem, can also be expressed as

$$g = g_1^{w_1} \times g_2^{w_2} \times \dots\dots \times g_k^{w_k} \qquad (5)$$

were

$$w_1 = \frac{n_1}{n_1 + n_2 + \dots\dots + n_k},$$
$$w_2 = \frac{n_2}{n_1 + n_2 + \dots\dots + n_k},$$
$$\dots\dots\dots\dots$$
$$w_k = \frac{n_k}{n_1 + n_2 + \dots\dots + n_k}.$$

This formula can sometimes be more convenient in computational works.

4. NUMERICAL EXAMPLE

Let us consider the data on sex ratio specifically data on number of female persons against 100,000 male persons in the eight states in North-East India in the years 1971, 1981, 1991, 2001 & 2011. Table – 1 has been prepared for these data computed from the primary data of number of male persons & number of female persons obtained from census report of India [18].

Table -1: Number of Females against 100,000 Males in the eight states in North-East India

State	Number of Female Persons in the Year				
	1971	1981	1991	2001	2011
Arunachal Pradesh	86088	86206	85925	89324	93823
Assam	89586	91033	92266	93478	95776
Manipur	98043	97079	95783	97417	98514
Meghalaya	94196	95378	95528	97164	98876
Mizoram	94580	91945	92144	93543	97574
Nagaland	87062	86321	88614	90045	93091
Sikkim	86259	83475	87803	87480	88992
Tripura	94269	94631	94453	94809	96007

Geometric mean of number of females against 100,000 males corresponding to the five years 1971, 1981, 1991, 2001 & 2011 has been computed by the formula given by equation (1) stated in the Definition the for each of the eight states. The values of the respective geometric means obtained have been shown in Table – 2.

Table -2: Geometric Means of Number of Females against 100,000 Males in the Eight States

State	Geometric Mean
Arunachal Pradesh	88221.700833629684222987836678275
Assam	92403.699741495728303276753811625
Manipur	97362.699133899541373946077257948
Meghalaya	96214.721340722491166124006843681
Mizoram	93935.116473616537679972830889016
Nagaland	88994.455098098488595069971296254
Sikkim	86781.267426899222764962743659332
Tripura	94831.825385123758593357331160095

Next, the geometric mean of number of females against 100,000 males in the region containing these eight states has been computed by the formula stated in the Theorem which has been found as

92269.622314059183456162683259718

Again, the geometric mean of number of females against 100,000 males in the region containing these eight states has also been found on computation from all the 40 observations in Table – 1 by the formula given by equation (1) stated in the Definition, as

92269.622314059183456162683259718

Thus, the two formulas yield the same result.

5. CONCLUSION

Such type of situation/problem arises in many studies where respective geometric means of several sets of observations obtained from independent studies are available but the primary observations are not available while it is required to find out the geometric mean of all the observations of the sets combined together. The formula derived here is a solution of this type of problem.

In this connection, it is to be mentioned that each of arithmetic mean, geometric mean & geometric mean is required to be computed in this type of situation. Of course, the formula for computing arithmetic mean in this situation is an established one while the formula for computing geometric mean has been derived here. However, the formulas for computing the other means in this type of situation are yet to be found out. This is a problem of research to be studied at this stage.

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