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Some Basic Properties of Standard Ratio of Data

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Abstract - A measure of dispersion of data of ratio type, **standard ratio (SR)** or **standard multiplicative deviation (SMD)** or **standard geometric deviation (SGD),** was developed on the basis of the ratios of the observations to their geometric mean. Some basic properties of this measure have been derived here due to their importance in theoretical development and practical applications. Derivations of the properties have been discussed in this article.

Keywords: Dispersion of Data, Measure, Standard Ratio, Basic Properties.

1. INTRODUCTION

Central tendency and Dispersion or variation [1 , 13 , 15 , 16 , 20] are two common characteristic of data which are closely related. Measures of dispersion [1 , 2 , 3 , 14 , 18 , 19] are numerical descriptions of the spread or variability of data. Measure of variation is closely related to measure of central tendency [16 , 17]. Measure of central tendency [1 , 3 , 5 , 14 , 21] is mostly based on measures of average specifically Pythagorean classical means and others means [4 , 5 , 12].

Several measures have already been developed for measuring the dispersion of data some of which are standard deviation , inter-quartile range, range , mean absolute difference, median absolute deviation, average absolute deviation, distance standard deviation etc. [1 , 2 , 3].

The existing measures of dispersion are defined on the basis of additive deviations of the observations from some arbitrary point and/or from some location parameter i.e. measure of central tendency of data [1 , 2 , 3 , 6 , 18 , 19]. Accordingly, these are not suitable for measuring variation in data of ratio type. Due to this reason, a measure of the dispersion of data, termed as **standard ratio (SR)** or **standard multiplicative deviation (SMD)** or **standard geometric deviation (SGD),** has recently been developed on the basis of the ratios of the observations to their geometric mean [6]. It is to be mentioned that several studies have already been done of basic properties of existing measures of central tendency and dispersion due to their importance [8 — 11]. In this study, some basic properties of this measure have been derived here due to their importance in theoretical development and practical applications. Derivations of the properties have been discussed in this article.

2. STANDARD RATIO

Standard Ratio of a Set of Numbers

Definition (2.1):

Let

$$A_1 , A_2 , \ldots\ldots , A_N$$

be N positive valued numbers whose geometric mean is G = G $(x_1, x_2, \ldots\ldots\ldots, x_n)$ which is defined by

$$G = G\,(A_1, A_2, \ldots\ldots\ldots, A_N) = (\prod_{i=1}^{N} A_i)^{\frac{1}{N}} \quad (2.1)$$

Then $\Omega = \Omega\,(A_1, A_2, \ldots\ldots\ldots, A_N)$, the standard ratio [6] of the numbers $A_1, A_2, \ldots\ldots\ldots, A_N$, is defined by

$$\Omega = \Omega\,(A_1, A_2, \ldots\ldots\ldots, A_N) = \{\prod_{i=1}^{n} \text{Ra}\,(G : A_i)\}^{\frac{1}{N}} \quad (2.2)$$

where

$\text{Ra}\,(G : A_i) = \frac{x_i}{G}$, if $A_i > G$,

$\text{Ra}\,(G : A_i) = \frac{G}{x_i}$, if $A_i < G$

& $\text{Ra}\,(G : A_i) = 1$, if $A_i = G$

Note (2.1):

(1) Definition of geometric mean given by equation (2.1) implies that

$$\text{Product of } A_1, A_2, \ldots\ldots\ldots, A_N = G\,N \quad (2.3)$$

(2) $\text{Ra}\,(G : A_i)$, defined above, is a ratio/fraction of A_i & G whose value is always greater than or equal to 1.

It measures the multiplicative deviation of A_i from G or equivalently of G from A_i .

Hence, $\text{Ra}\,(G : A_i)$ can be regarded as the Absolute Multiplicative Deviation (AMD) of A_i from G.

Therefore, equation (2.1) given in the Definition (2.1) implies that

Product of the AMDs of $A_1, A_2, \ldots\ldots\ldots, A_N$ from their geometric mean = Ω N (2.4)

Standard Ratio of a Variable

Definition (2.2):

Let X be a variable which assumes the n positive values

$$x_1, x_2, \ldots\ldots\ldots, x_n$$

whose geometric mean (GM) is g = G (X) i.e.

$$g = G\,(X) = \{\prod_{i=1}^{n} x_i)\}^{\frac{1}{n}} \quad (2.5)$$

By the similar logic as applied in forming Definition (2.1), the standard ratio of X, denoted by $\Omega = \Omega$ (X), can be defined by

$$\Omega = \Omega\,(X) = \{\prod_{i=1}^{n} \text{Ra}\,(g : x_i)\}^{\frac{1}{n}} \quad (2.6)$$

where

$$\text{Ra}\,(g : x_i) = \frac{x_i}{g}, \text{ if } x_i > g,$$

$$\text{Ra}\,(g : x_i) = \frac{g}{x_i}, \text{ if } x_i < g$$

$$\& \quad \text{Ra}\,(g : x_i) = 1, \text{ if } x_i = g$$

Note (2.2):

(1) Definition of geometric mean given by equation (2.1) implies that

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Product of the values assumed by X = g^n (2.7)

(2) Ra $(g : x_i)$, defined above, is an **improper fraction** of x_i & G whose value is always greater than or equal to 1.

It measures the multiplicative deviation of x_i from g or equivalently of g from x_i.

Hence, Ra $(g : x_i)$ can be regarded as the **Absolute Multiplicative Deviation (AMD)** of x_i from g.

Therefore, equation (2.6) given in the **Definition (2.2)** implies that

Product of **AMDs** of the values assumed by X from their geometric mean = Ω^n (2.8)

3. SOME BASIC PROPERTIES

Property-1 (Improper Decimal Property): **Standard ratio** of a variable is an improper decimal number i.e. for any variable X,

$$\Omega(X) > 1$$

Proof

This follows from the definition of Ω (X).

Note (2.3):

Ω (X) = 1 if and only if X is a constant.

Property-2 (Invariance Property): For any non-zero constant c,

$$\Omega(cX) = \Omega(X)$$

Proof

This follows from the fact that

G(cX) = $(\prod_{i=1}^{n} c\, x_i)^{1/n}$ = c.$(\prod_{i=1}^{n} x_i)^{1/n}$ = c.G(X)

which implies, Ra $(g : cx_i)$ = Ra $(g : x_i)$

i.e. $\{ \prod_{i=1}^{n} \text{Ra}\,(g : cx_i) \}^{\frac{1}{n}} = \{ \prod_{i=1}^{n} \text{Ra}\,(g : x_i) \}^{\frac{1}{n}}$

i.e. Ω (cX) = Ω (X)

In particular, putting c = – 1, one can obtain the result

$$\Omega(-X) = \Omega(X)$$

Remark: This property of standard ratio can be interpreted as follows:

"Standard ratio is invariant of change of scale."

Property-3 (Exponent Property): For real non-zero m,

$$\Omega(X^m) = \{\Omega(X)\}^m$$

Proof:

If g is the **geometric mean** of X then as a consequence of multiplicative property of **geometric mean** [7 , 9 , 11, gm is the geometric mean of X^m.

Now,

$$\frac{X}{g} > 1 \Rightarrow \frac{X^m}{g^m} > 1 \quad \& \quad \frac{g}{X} > 1 \quad \Rightarrow \frac{g^m}{X^m} > 1$$

as well as

$$\frac{X^m}{g^m} > 1 \Rightarrow \frac{X}{g} > 1 \quad \& \quad \frac{g^m}{X^m} > 1 \Rightarrow \frac{g}{X} > 1$$

Moreover,

$$\frac{X^m}{g^m} = \{\frac{X}{g}\}^m \quad \& \quad \frac{g^m}{X^m} = \{\frac{g}{X}\}^m ,$$

$$\{\frac{X^m}{g^m}\}^{\frac{1}{m}} = \frac{X}{g} \quad \& \quad \{\frac{g^m}{X^m}\}^{\frac{1}{m}} = \frac{g}{X}$$

Since Ra $(g : x_i)$ is the improper fraction of x_i & g and Ra $(g^m : {x_i}^m)$ is the improper fraction of ${x_i}^m$ & g^m ,

therefore, Ra $(g^m : {x_i}^m) = \{\text{Ra}\,(g : x_i)\}^m$

which implies, $\prod_{i=1}^{n}$ Ra $(g^m : {x_i}^m) = \{\prod_{i=1}^{n} \{\text{Ra}\,(g : x_i)\}^m$

i.e. $\{\prod_{i=1}^{n}$ Ra $(g^m : {x_i}^m)\}^{\frac{1}{n}} = [\{\prod_{i=1}^{n} \{\text{Ra}\,(g : x_i)\}^m]^{\frac{1}{n}}$

i.e. $\{\prod_{i=1}^{n}$ Ra $(g^m : {x_i}^m)\}^{\frac{1}{n}} = [\{\prod_{i=1}^{n} \{\text{Ra}\,(g : x_i)\}^{\frac{1}{n}}]^m$

Hence, $\Omega\,(X^m) = \{\Omega\,(X)\}^m$

Corollary:

In particular putting m = – 1 in the this result, one can obtain the following result:

$$\Omega\,(\frac{1}{X}) = \frac{1}{\Omega\,(X)}$$

Property-4 (Multiplicative Property): For two positive valued variables X & Y,

$$\Omega\,(XY) = \Omega\,(X).\Omega\,(Y)$$

In general, if

$$X_1 , X_2 , \ldots\ldots\ldots , X_k$$

are k positive valued variables then

$$\Omega\,(X_1 . X_2 . \ldots\ldots\ldots . X_k) = \Omega\,(X_1) . \Omega\,(X_2) . \ldots\ldots\ldots . \Omega\,(X_k)$$

Proof:

Suppose, X assumes the n positive values

$$x_1 , x_2 , \dots\dots\dots , x_n$$

having geometric mean g_x

& Y assumes the p positive values

$$y_1 , y_2 , \dots\dots\dots , y_p$$

having geometric mean g_y .

If g_{xy} is the **geometric mean** of the variable XY then by multiplicative property of **geometric mean** [7 , 9 , 11],

$$g_{xy} = g_x g_y$$

so that

$$\text{Ra} \left(g_{xy} : x_i y_j \right) = \text{Ra} \left(g_x g_y : x_i y_j \right)$$

Now,

$$\frac{x_i y_j}{g_{xy}} = \frac{x_i y_j}{g_x g_y} = \frac{x_i}{g_x} \cdot \frac{y_j}{g_y}$$

which implies,

Improper fraction of $x_i y_j$ & g_{xy}

= (**Improper fraction** of x_i & g_x) × (**Improper fraction** of y_j & g_y)

Therefore,

$\text{Ra} \left(g_{xy} : x_i y_j \right) = \text{Ra} \left(g_x : x_i \right). \text{Ra} \left(g_y : y_j \right)$

which implies,

$\{ \prod_{i=1}^{n} \prod_{j=1}^{p} \text{Ra}(g_{xy} : x_i y_j) \}^{\frac{1}{np}} = \{ \prod_{i=1}^{n} \text{Ra}(g_x : x_i)\}^{\frac{1}{n}} \{ \prod_{j=1}^{p} \text{Ra}(g_y : y_{j_i})\}^{\frac{1}{p}}$

Hence,

Ω (XY) = Ω (X).Ω (Y)

Now, for three positive valued variables

X_1 , X_2 , X_3 ,

it is obtained that

$\Omega \left(X_1 X_2 X_3\right) = \Omega \{(X_1 X_2). X_3)$

$= \Omega \left(X_1 X_2\right). \Omega \left(X_3\right)$

$= \Omega \left(X_1 \Omega \left(X_2\right). \Omega \left(X_3\right)\right.$

i.e. $\Omega \left(X_1 X_2 X_3\right) = \Omega \left(X_1 \Omega \left(X_2\right). \Omega \left(X_3\right)\right.$

By similar logic, it is obtained for four positive valued variables

$$X_1 , X_2 , X_3 , X_4$$

that

$$\Omega\,(X_1X_2\;X_3X_4) \;=\; \Omega\,(X_1\Omega\,(X_2).\Omega\,(X_3).\,\Omega\,(X_4)$$

Continuing the process, it is obtained that for k positive valued variables

$$X_1\,,X_2\,,\;\ldots\ldots\ldots\;,\;X_k\,,$$

$$\Omega\,(X_1\,.\,X_2\,.\;\ldots\ldots\ldots\;.\;X_k) \;=\; \Omega\,(\,X_1).\Omega\,(\,X_2)\,.\;\ldots\ldots\ldots\;.\,\Omega\,(\,X_k)$$

Property-5 (Quotient Property): For two positive valued variables X & Y,

$$\Omega\,(\tfrac{X}{Y}) \;=\; \frac{\Omega\,(X)}{\Omega\,(Y)}$$

In general for positive valued variables

$$X_1\,,X_2\,,\;\ldots\ldots\ldots\;,X_k \quad \& \quad Y_1\,,Y_2\,,\;\ldots\ldots\ldots\;,Y_s\;,$$

$$\Omega\,(\frac{X_1\,.X_2\,.\;\ldots\ldots\ldots\;.X_k}{Y_1\,.Y_2\,.\;\ldots\ldots\ldots\;.Y_s}) \;=\; \frac{\Omega\,(X_1).\Omega\,(X_2)\,.\;\ldots\ldots\ldots\;.\Omega\,(X_k)}{\Omega\,(Y_1).\Omega\,(Y_2)\,.\;\ldots\ldots\ldots\;.\Omega\,(Y_s)}$$

Proof:

We have

$$\Omega\,(\tfrac{X}{Y}) \;=\; \Omega\,\{(X.(\tfrac{1}{Y})\}$$

$$=\; \Omega\,(X).\,\Omega\,(\tfrac{1}{Y}) \quad , \text{ by multiplicative property}$$

$$=\; \Omega\,(X).\frac{1}{\Omega\,(Y)} \quad , \text{ by the corollary of exponent property}$$

$$=\; \frac{\Omega\,(X)}{\Omega\,(Y)}$$

Similarly, applying **multiplicative property** and the **corollary** of **exponent property**, it is obtained that

$$\Omega\,(\frac{X_1\,.X_2\,.\;\ldots\ldots\ldots\;.X_k}{Y_1\,.Y_2\,.\;\ldots\ldots\ldots\;.Y_s})$$

$$=\; \Omega\,(X_1.\,X_2\,.\;\ldots\ldots\ldots\;.X_k)\,.\,\Omega\,(\frac{1}{Y_1\,.Y_2\,.\;\ldots\ldots\ldots\;.Y_s})$$

$$=\; \Omega\,(X_1.\,X_2\,.\;\ldots\ldots\ldots\;.X_k)\,.\,\Omega\,(\tfrac{1}{Y_1})\;\Omega\,(\tfrac{1}{Y_2}).\;\ldots\ldots\;.\,\Omega\,(\tfrac{1}{Y_s})$$

$$=\; \Omega\,(\,X_1).\Omega\,(\,X_2)\,.\;\ldots\ldots\ldots\;.\,\Omega\,(\,X_k)\,.\,\frac{1}{\Omega\,(Y_1)}\,\frac{1}{\Omega\,(Y_2)}.\;\ldots\ldots\;.\,\frac{1}{\Omega\,(Y_s)}$$

$$=\; \frac{\Omega\,(X_1).\Omega\,(X_2)\,.\;\ldots\ldots\ldots\;.\Omega\,(X_k)}{\Omega\,(Y_1).\Omega\,(Y_2)\,.\;\ldots\ldots\ldots\;.\Omega\,(Y_s)}$$

4. CONCLUSION

The basic properties, derived above, can be useful in identifying/deriving more properties of the measure of dispersion derived here.

The properties developed here can also be helpful in calculation the value of this measure dispersion from data having large valued numerical observations.

In this article, only five properties of standard ratio have been identified while there may be more properties, which carry significance, to be satisfied by this measure. Accordingly, there is necessity of those unknown properties to be identified for the interest of establishing it as a more important.

REFERENCES

[1] Ali Zulfiqar; Bhaskar, S Bala & Sudheesh, K (2019): "Descriptive Statistics: Measures of Central Tendency, Dispersion, Correlation and Regression", Airway, 2(3), 120 – 125. DOI: 10.4103/ARWY.ARWY_37_19 .

[2] Anderson T. W. & Finn J. D. (1996): "Measures of Variability. In: The New Statistical Analysis of Data", Springer, New York, NY. https://doi.org/10.1007/978-1-4612-4000-6_4 .

[3] Argyrous G. (1997): "Measures of Central Tendency and Measures of Dispersion", In: Statistics for Social Research, Palgrave, London. https://doi.org/10.1007/978-1-349-14777-9_4 .

[4] Coggeshall F. (1886): "The Arithmetic, Geometric, and Harmonic Means", The Quarterly Journal of Economics, 1(1), 83–86. https://doi.org/10.2307/1883111 . https://www.jstor.org/stable/1883111 .

[5] Dhritikesh Chakrabarty (2021): "Measuremental Data: Seven Measures of Central Tendency", International Journal of Electronics and Applied Research (ISSN : 2395 – 0064), 8(1), 15 – 24. http://eses.net.in/online_journal.html .

[6] Dhritikesh Chakrabarty (2024): "Measure of Variation in Data of Ratio Type: Standard Multiplicative Deviation", Partners Universal International Research Journal (PUIRJ), (ISSN: 2583-5602), 03(03), 110 – 119. www.puirj.com . DOI:10.5281/zenodo.13827583.

[7] Dhritikesh Chakrabarty (2024): "Multiplicative Property of Geometric Mean", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN: 2350 – 0328), 11(11), 22534 – 22541. www.ijarset.com . https://www.researchgate.net/publication/386284830_Multiplicative_Property_of_Geometric_Mean .

[8] Dhritikesh Chakrabarty (2024): "Additive Property of Harmonic Mean from that of Arithmetic Mean", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN: 2350 – 0328), 11(12), 22668 – 227676. www.ijarset.com . https://www.researchgate.net/publication/387559146_Additive_Property_of_Harmonic_Mean_from_that_of_Arithmetic_Mean.

[9] Dhritikesh Chakrabarty (2025): "Multiplicative Property of Geometric Mean: Second Proof", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN: 2350 – 0328), 12(1), 22771 – 22778. www.ijarset.com. https://www.researchgate.net/publication/388555515_Multiplicative_Property_of_Geometric_Mean_Second_Proof.

[10] Dhritikesh Chakrabarty (2025): "Combined Set of Several Sets of Observations: Harmonic Mean", Partners Universal International Innovation Journal (PUIIJ), 3(1), 49 – 53. www.puiij.com . DOI:10.5281/zenodo.14949601. https://www.researchgate.net/publication/389541894_Combined_Set_of_Several_Sets_of_Observations_Harmonic_Mean .

[11] Dhritikesh Chakrabarty (2025): "Multiplicative Property of Geometric Mean: Another Proof", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN: 2350 – 0328), 12(2), 22888 – 22895. www.ijarset.com. https://www.researchgate.net/publication/389441108_Multiplicative_Property_of_Geometric_Mean_Aother_Proof

[12] Geoffrey Hunter (2020): "The Three Classical Pythagorean Means", mbedded.ninja, https://blog.mbedded.ninja › Mathematics › Statistics .

[13] Herbert F. Weisberg (1992): "Central Tendency and Variability, Series: Quantitative Applications in the Social Sciences", Issue 83, ", Chapter- 4, 46 – 75, Sage Publication, London.

[14] Jain Sharad K. & Vijay P. Singh (2019): "Key Statistical Measures of Data", Chap. 18.2 in Engineering Hydrology: An Introduction to Processes, Analysis, and Modeling, McGraw-Hill Education, New York. https://www.accessengineeringlibrary.com/content/book/9781259641978/toc-chapter/chapter18/section/section6

[15] John H. Mc Donald (2024): "Statistics of Dispersion" Section-3.2, Statistics LibreTexts , https://stats.libretexts.org .

[16] Kelly Ivan W. & James E. Beamer (1986): "Central Tendency and Dispersion: The Essential Union", The Mathematics Teacher, 79(1), 59 – 65. JSTOR, http://www.jstor.org/stable/27964757 . Accessed 9 June 2024.
[17] Malakar I. M. (2023): "Conceptualizing Central Tendency and Dispersion in Applied Statistics", Cognition, 5(1), 50 – 62. https://doi.org/10.3126/cognition.v5i1.55408 .
[18] Moore P. G. (2010): "Principles of Statistical Techniques - Measures of Dispersion" Chapter-7, Cambridge University Press.
[19] Murray R. Spiegel & Larry J. Stephens(2018): The Standard Deviation and Other Measures of Dispersion", In the book "Schaum's Outline of Statistics" Chapter-4, ISBN: 9781260011463, McGraw Hill. https://www.accessscience.com › chapter › chapter4.
[20] Weisberg H. F. (1992): "Central Tendency and Variability", Sage University Paper Series on Quantitative Applications in the Social Sciences, ISBN 0-8039-4007-6 pp.2.
[21] Williams R. B. G.(1984):"Measures of Central Tendency", Introduction to Statistics for Geographers and Earth Scientist, Soft cover ISBN978-0-333-35275-5, eBook ISBN978-1-349-06815-9 , Palgrave, London, 51 – 60.