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Extended Definition of Statistical Probability: Estimate of Probability Distribution of Rainy Days in Southern Part of India

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Abstract - Recently the statistical definition of probability introduced by von Mises, which was based on the outcomes of actually performed experimentation, has been extended to the situation where outcomes of the trials happened automatically. This extended definition of probability has been applied in estimating probability distribution of rainy days in each of the 12 months at four stations in southern part of India namely Bangalore, Chennai, Hyderabad & Trivandrum with a view of obtaining a picture of tendency of rainfall there. This article presents the findings of estimates of obtained in the study. It has been found from the study that at each of the four stations there does not exist any month which is certainly non-rainy and there exists months which are certain rainy.

Keywords: Statistical probability, automatically happened trials, rainy day, probability distribution.

1. INTRODUCTION

Probability is the basis of analysis of data when analyzed statistically. [39 , 40]. The definition of probability was formulated by different approaches in different times. The first approach which was a biased, subjective and unscientific one was introduced by Thomas Bayes [2]. The second one, known as Intuitive Approach which was also not based on scientific logic was introduced by Koopman & Savage [37 , 38 , 43 , 44 , 45]. The third one is the Classical Approach which was framed of by James Bernoull [3 , 8 , 11 , 12]. After that the Empirical Approach (which is also known as Relative Frequency Approach or Statistical Approach) was developed by von Mises [8 , 10 , 31 , 43 , 48 , 49 , 50 , 51]. The modern approach to probability known as Axiomatic Approach was developed by Bernstein & Kolmogorov [3 , 4 , 5 , 35 , 36]. Recently, another approach, known as Theoretical Approach has been developed by Chakrabarty [7 , 10 – 17 , 22 , 23]. Each of the approaches other than the subjective approach and the intuitive approach is based on scientific logic.

In each of the four scientific, probability is defined or determined on the basis of random experiment either performing the actual experimentation or prior to performing it. In many real situations, experimentation need not be and/or cannot be performed but is automatically performed resulting in available outcomes. Recently the statistical definition of probability introduced by von Mises, which was based on the outcomes of actually performed experimentation, has been extended to the situation where outcomes of the trials happened automatically [27 , 29 , 30]. This extended definition of probability has been applied in estimating probability distribution of rainy days in each of the 12 months at four stations in southern part of India namely Bangalore , Chennai , Hyderabad & Trivandrum with a view of obtaining a picture of tendency of rainfall a the stations. It is to be mentioned that several studies have been done so far on tendency of rainfall [1 , 6 , 18 – 20, 27 , 32 – 34 , 41 , 42 , 45 – 47 , 52 – 54]. However, these have been by descriptive statistical methods [21, 24 – 26 , 28]. Probabilistic approach has hardly been applied in the studies. That is why the

probabilistic approach has been applied in this study. This article presents the findings of estimates of obtained in the study.

2. AUTOMATIC TRIALS AND PROBABILITY

The following fact is a consequence of the statistical definition of probability introduced by von Mises [8 , 10 , 31 , 48 , 49 , 50 , 51]:

If the random performing of a trial is repeated N times under identical homogenous condition and if out of the N repetitions of the trail an event E occurs n times then the probability of occurrence of the event E, denoted by P(E), is a number towards which the ratio $\frac{n}{N}$ approaches as N becomes larger i.e.

$\frac{n}{N} \rightarrow$ P(E) as $N \rightarrow \infty$

i.e. P(E) is the limiting value of $\frac{n}{N}$ as N becomes larger and larger.

Conversely,

if the random performing of a trial is repeated N times under identical homogenous condition then the probability of occurrence of the event E, denoted by P(E), is a number such that the number of occurrence n of the event E out of the N repetitions of the trial approaches N.P(E) as N becomes larger i.e.

$n \rightarrow$ N.P(E) as $N \rightarrow \infty$

This definition states that the number of occurrence of the event E out of N repetitions of the trial can be approximated by N.P(E) provided N is large.

The following fundamental properties/ results of probability can be obtained from the definition:

(1) The probability of occurrence (or of non-occurrence) of an event lies within (0 , 1).

(2) If the probability of occurrence of an event is 1, the occurrence of the event is certain.

(3) If the probability of occurrence of an event is 0, the non-occurrence of the event is certain.

In all other cases, neither the occurrence nor the non-occurrence of the event is certain.

(4) The sum of the probability of occurrence an event and the probability of its non-occurrence event is 1. This means, it is certain that an event either occurs or does not occur.

(5) The probability of occurrence of any of the all possible outcomes of a trial is 1.

(6) The probability of occurrence of either of a finite number of events is the sum of the individual probabilities of the occurrences of the events.

(7) The probability of occurrence of either of a denumerable number of events is the sum of the individual probabilities of the occurrences of the events provided the sum is convergent.

2.1. Rainy Days and Probability

Probability of Number of Rainy Days

Suppose that E is an event that denotes occurrence of r rainy days in a month.

Consider the observations on happenings of rainfall in the month on a number of years (say N years) i.e. on N repetitions of the happenings.

Since the phenomenon has happened naturally, it is free from error that occurs due to performing of experiment.

Moreover, the natural happening of the phenomenon can be thought of as the performing of experiment on rainfall not by human but by nature.

If out of N repetitions the event E occurs N(E) times then the probability of occurrence of the event E, denoted by P(E), can be defined by the number towards which the ratio $\frac{n}{N}$

approaches as N becomes larger i.e.

$$\frac{n}{N} \rightarrow P(E) \quad \text{as} \quad N \rightarrow \infty$$

i.e. P(E) is the limiting value of $\frac{n}{N}$ as N becomes larger and larger.

Note:

For finite N i.e. for sample of finite size, the value of this ratio may not be equal to the actual value of the probability P(E). However, it can be regarded as estimator of P(E) due to the above limiting property.

3. PROBABILITY OF RAINY DAYS AT BANGALORE, CHENNAI, HYDERABAD, & TRIVANDRUM

The definition of probability based on the data on automatically happened outcomes has been applied in probability distribution of number of rainy days occurred in each of the 12 months at four capital cities of southern part of India namely Bangalore , Chennai , Hyderabad & Trivandrum on the basis of data from the year 1969 onwards collected from Indian Meteorological Department.

The number of rainy days considered here are the point values as well as the interval values (of length 3 and/or of 5) depending upon the data on the station and on the month.

The definition of probability of number of rainy days, as explained above, has been applied in estimating the probabilities of possible number of rainy days in each of the 12 months at the four stations. The estimated values obtained have been presented in Table – 5.1, Table – 5.2, Table – 5.3 & Table – 5.4 respectively.

4. RESULT AND DISCUSSION

Some special information obtained from the findings of this study are as follows:

(1) None of the four stations namely Bangalore, Chennai, Hyderabad & Trivandrum is completely free from rainfall in any month. Of course,

at Bangalore, the period January – April is more likely to be non-rainy,

at Chennai, the period January – April is more likely to be non-rainy,

at Hyderabad, the period January – March is more likely to be non-rainy.

(2) The periods May – October, June – November, June – October & April – November are certain to be rainy at Bangalore, Chennai, Hyderabad & Trivandrum respectively.

(3) Chance of rainfall is very less

in the months April, November & December at Bangalore,

in the months May & December at Chennai,

in the months April, May, November & December at Hyderabad & in the months January, February, March & December at Trivandrum respectively.

Findings of this study leads to a conclusion that the probability defined for outcomes happened automatically can suitably applied in estimating probability distribution of number of rainy days. Similar method can be used in at other places/stations not considered in this study. Thus the researchers can attempt to estimate probability distributions of number of rainy days at the other places of the globe by the application of the definition of probability based on automatically happened outcomes. Findings of such study will be very important and useful information in the research study on global environment.

One more point to be noted is that in this study attempt has been made on estimating probability occurrence of rainfall in (terms of rainy days) with a view of obtaining a picture of tendency of rainfall there.. The picture becomes clearer if the expected number of rainy days can be obtained. Accordingly, one necessary work at this stage is to attempt to determine mathematical expectation of number of rainy days with the help of probability defined for outcomes happened automatically.

5. TABLES OF FINDINGS

Table -5.1 : Estimated Probability Distribution of Number of Rainy Days at BANGALORE

January		**February**		**March**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
0	0.875	0	0.71875	0	0.6875
1	0.09375	1	0.15625	1	0.09375
2	0	2	0.0625	2	0.09375
3	0.03125	3	0.0625	3	0.09375
				4	0.03125
April		**May**		**June**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
0	0.0625	3	0.0625	2	0.09375
1	0.21875	4 – 8	0.71875	3	0.0625
2	0.125	9 – 11	0.21875	4 – 8	0.59375
3 – 7	0.59375			9 – 11	0.25

July		August		September	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
3 – 5	0.3125	6 – 10	0.59375	4 – 8	0.34375
6 – 8	0.40625	11 – 15	0.375	9 – 13	0.46875
9 – 11	0.15625	16 – 17	0	14 – 16	0.15625
12 – 14	0.125	18	0.03125	17	0.03125
October		**November**		**December**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
2	0.03125	0	0.0625	0	0.3125
3 – 5	0.15625	1	0.0625	1	0.25
6 – 8	0.3125	2	0.15625	2	0.125
9 – 11	0.40625	3 – 7	0.625	3	0.125
12 – 14	0.09375	8 – 10	0.09375	4 – 6	0.1875

Table -5.2 : Estimated Probability Distribution of Number of Rainy Days at CHENNAI

January		February		March	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
0	0.53333	0	0.8	0	0.76667
1	0.13333	1	0.066667	1	0.16667
2	0.1	2	0.1	2	0
3	0.13333	3 – 7	0	3	0.06667
4	0.066667	8	0.033333		
5 – 6	0				
7	0.033333				
April		**May**		**June**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
0	0.6	0	0.3	1	0.1
1	0.13333	1	0.36667	2	0.13333

2	0.16667	2	0.06667	3	0.13333
3	0.066667	3 – 5	0.26667	4 – 6	0.4
4	0.03333			7 – 9	0.23333
July		**August**		**September**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
1	0.06667	1	0.03333	2	0.06452
2	0.03333	2	0.066667	3	0.03226
3 – 7	0.43333	3 – 7	0.366667	4 – 8	0.51613
8 – 12	0.46667	8 – 12	0.366667	9 – 13	0.38709
		13 – 15	0.16667		
October		**November**		**December**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
2 – 6	0.23333	4 – 8	0.33333	0	0.1
7 – 11	0.36667	9 – 13	0.533333	1	0.06667
12 – 16	0.36667	14 – 18	0.1	2	0.1
17 – 20	0	19 – 20	0	3	0.13333
21	0.03333	21	0.03333	4 – 8	0.43333
				9 – 13	0.1
				14	0.03333
				15 – 17	0
				18	0.03333

Table -5.3 : Estimated Probability Distribution of Number of Rainy Days at HYDERABAD

January		**February**		**March**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
0	0.63333	0	0.73333	0	0.6
1	0.26667	1	0.16667	1	0.23333
2	0.06667	2	0.06667	2	0.16667
3	0	3	0		

4	0.033333	4	0.03333		
April		**May**		**June**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
0	0.23333	0	0.2	1	0.03333
1	0.36667	1	0.2	2 – 3	0
2	0.2	2	0.13333	4 – 8	0.66667
3	0.13333	3 – 7	0.43333	9 – 13	0.26667
4	0.06667	8	0.03333	14	0.03333
July		**August**		**September**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
3 – 7	0.31034	3 – 7	0.16667	1	0.03333
8 – 12	0.44828	8 – 12	0.53333	2	0
13 – 17	0.24138	13 – 17	0.26667	3 – 7	0.43333
		18	0.03333	8 – 12	0.46667
				13 – 15	0.06667
October		**November**		**December**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
1	0.1	0	0.33333	0	0.7
2	0.13333	1	0.13333	1	0.23333
3 – 7	0.5	2	0.13333	2	0.06667
8 – 12	0.2	3	0.23333		
13 – 15	0.06667	4 – 6	0.16667		

Table -5.4 : Estimated Probability Distribution of Number of Rainy Days at TRIVANDRUM

January		**February**		**March**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
0	0.46667	0	0.43333	0	0.2
1	0.33333	1	0.16667	1	0.2

2	0.1	2	0.23333	2	0.23333
3	0.03333	3	0.06667	3	0.166667
4 – 6	0.066667	4	0.03333	4 – 6	0.166667
		5 – 6	0	7 – 9	0
		7	0.06667	10	0.03333
April		**May**		**June**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
1	0.03333	3 – 7	0.3	4 – 8	0.06667
2	0.06667	8 – 12	0.5	9 – 13	0.16667
3	0	13 – 17	0.13333	14 – 18	0.36667
4 – 8	0.66667	18 – 20	0.06667	19 – 23	0.33333
9 – 13	0.23333			24– 27	0.06667
July		**August**		**September**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
6	0.03333	2	0.03333	1	0.03333
7 – 8	0	3 – 7	0.2	2 – 6	0.3
9 – 13	0.5	8 – 12	0.53333	7 – 11	0.43333
14 – 18	0.4	13 – 17	0.23333	12 – 16	0.16667
19	0.03333			17 – 19	0.06667
22	0.03333				
October		**November**		**December**	
Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence	Number of Rainy Days	Probability of occurrence
1	0.03333	2	0.03333	0	0.06667
2 – 3	0	3 – 4	0	1	0.03333
4 – 8	0.23333	5 – 9	0.5	2	0.33333
9 – 11	0.16667	10 – 12	0.3	3 – 7	0.43333
12 – 14	0.3	13 – 15	0.1	8 – 12	0.13333
15 – 17	0.2	16 – 18	0.06667		
18 – 20	0.06667				

REFERENCES

[1] Alessia Flammini, Jacopo Dari, Corrado Corradini, Carla Saltalippi, Renato Morbidelli (2022): "Areal reduction factor estimate for extreme rainfall events", Rainfall, 10.1016/B978-0-12-822544-8.00014-7, 285 – 306.

[2] Antonio Pasculli, Jacopo Cinosi, Laura Turconi, Nicola Sciarra (2021): "Learning Case Study of a Shallow-Water Model to Assess an Early-Warning System for Fast Alpine Muddy-Debris-Flow", Water, 13(6), 750. 10.3390/w13060750.

[3] Breiman L. (1968): "Probability", Addison-Wesley, Reading, MA.

[4] Bruno Sansó, Lelys Guenni (2004: "A Bayesian approach to compare observed rainfall data to deterministic simulations", Environmetrics, 15(6), 597 – 612. 10.1002/env.660.

[5] Christoffer B. Andersen, Daniel B. Wright, Søren Thorndahl (2022): "Sub-Hourly to Daily Rainfall Intensity-Duration-Frequency Estimation Using Stochastic Storm Transposition and Discontinuous Radar Data", Water, 14(24), 4013. 10.3390/w14244013.

[6] Caloiero T., Coscarelli R. & Pellicone G. (2021), "Trend Analysis of Rainfall Using Gridded Data over a Region of Southern Italy", Water 2021, 13, 2271. https://doi.org/10.3390/w13162271.

[7] Dhritikesh Chakrabarty (2004) : " A Theoretical Definition of Probability Based on Common Sense ", Bulletin of Pure and Applied Sciences - E (ISSN : 0970 -6577), 23E(2), 343 – 349. https://www.researchgate.net/publication/265315010_A_theoretical_definition_of_probability_based_on_common_sense?

[8] Dhritikesh Chakrabarty (2005) : " Probability : Link Between the Classical Definition and the Empirical Definition ", J. Ass. Sc. Soc. (ISSN : 0587-1921), 45(June), 13 – 18. https://www.researchgate.net/publication/322759139_Probability_Link_Between_the_Classical_Definition_and_the_Empirical_Definition?

[9] Dhritikesh Chakrabarty (2006) : " Non-Equally Likely Outcomes : The Classical Definition of Probability ", Bulletin of Pure and Applied Sciences - E (ISSN : 0970 -6577), 25E(2), 471 – 481. https://www.researchgate.net/publication/264911628_Non-equally_likely_outcomes_the_classical_definition_of_probability?

[10] Dhritikesh Chakrabarty (2007) : " Empirical Definition of Probability : Special Case of Its Theoretical Definition ", Int. J. Agricult. Stat. Sci., (ISSN : 0973 - 1903), 3(1), 261 – 267. https://www.researchgate.net/publication/267078755_Empirical_definition_of_probability_Special_case_of_its_theoretical_definition?

[11] Dhritikesh Chakrabarty (2008) : "Bernoulli's Definition of Probability : Special Case of Its Chakrabarty's Definition ", Int. J. Agricult. Stat. Sci., (ISSN : 0973 - 1903), 4(1), 23 – 27. https://www.researchgate.net/publication/265477707_Bernoulli's_definition_of_probability_special_case_of_its_Chakrabarty's_definition?

[12] Dhritikesh Chakrabarty (2009) : " Probability: Chakrabarty's Definition from its Classical Definition ", Int. J. Agricult. Stat. Sci., (ISSN : 0973 - 1903), 5(1), 181 – 187. https://www.researchgate.net/publication/299265157_PROBABILITY_CHAKRABARTY'S_DEFINITION_FROM_ITS_CLASSICAL_DEFINITION?

[13] Dhritikesh Chakrabarty (2010) : "A Method of Determining the Value of Probability", Int. J. for Statiscians, (ISSN : 2077 – 480X), 1(1), 5 – 7. https://www.researchgate.net/publication/322758586_A_Method_of_Determining_the_Value_of_Probability?

[14] Dhritikesh Chakrabarty (2010b) : " Probability As The Maximum Occurrence of Relative Frequency ", Arya Bhatta Journal of Mathematics & Informatics., 2 (2), 339 – 344. www.abjni.com .

[15] Dhritikesh Chakrabarty (2010) : "Chakrabarty's Definition of Probability: Proper Randomness of Fisher and Yates Random Number Table ", Int. J. Agricult. Stat. Sci., 6 (2), (ISSN : 0973 – 1903), 461 – 469. https://www.researchgate.net/publication/289843999_Chakrabarty's_definition_of_probability_Proper_randomness_of_fisher_and_yates_random_number_table?

[16] Dhritikesh Chakrabarty (2010) : "Chakrabarty's Definition of Probability: Additive and Multiplicative Laws ", Bulletin of Pure and Applied Sciences - E (Print ISSN : 0970 -6577), 29E(2), 265 – 274. Online published on 22 February, 2013 (Online ISSN: 2320-3226). https://www.researchgate.net/publication/267991650_Chakrabarty's_definition_of_probability_additive_and_multiplicative_laws?

[17] Dhritikesh Chakrabarty (2011): "Probability in Ideal Situation and in Practical Situation", Arya Bhatta J. Math. & Info. , 3 (1), 161 – 168. www.abjni.com .

[18] Dhritikesh Chakrabarty (2014) : "Natural Limits of Annual Total Rainfall in the Context of India ", Int. J. Agricult. Stat. Sci., 10(1), (ISSN : 0973 – 1903), 105 – 109. https://www.researchgate.net/publication/296323600_Natural_limits_of_annual_total_rainfall_in_the_context_of_India?

[19] Dhritikesh Chakrabarty (2019): "Significance of Change of Rainfall: Confidence Interval of Annual Total Rainfall", Journal of Chemical, Biological and Physical Sciences (E- ISSN : 2249 – 1929), Sec. C, 9(3), 151 – 166. www.jcbsc.org . DOI: 10.24214/jcbps.C.9.

[20] Dhritikesh Chakrabarty (2021): "Annual Total Rainfall in India: Confidence Interval and Significance of Change", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN : 2350 – 0328), 8(11), 18540 – 18550. www.ijarset.com .

[21] Dhritikesh Chakrabarty (2022): "Integral Valued Numerical Data: Measure of Central Tendency", Partners Universal International Research Journal (PUIRJ), 01(03), 74 – 82. www.puirj.com . DOI:10.5281/zenodo.7123662 .

[22] Dhritikesh Chakrabarty (2022): "Latest Definition of Probability: Link with Its Earlier Definitions", Unpublished Research Paper, Uploaded in Research Gate on May 15, 2022. DOI: 10.13140/RG.2.2.28013.15844. https://www.researchgate.net/publication/360612422_Latest_Definition_of_Probability_Link_with_Its_Earlier_Definitions?

[23] Dhritikesh Chakrabarty (2022): "Probability in Practically Ideal Situation", Unpublished Research Paper, Uploaded in Research Gate on September 23, 2022. DOI: 10.13140/RG.2.2.23818.85443. https://www.researchgate.net/publication/363771306_PROBABILITY_IN_PRACTICALLY_IDEAL_SITUATION?

[24] Dhritikesh Chakrabarty (2022): "Determination of Tendency of Rainfall at Delhi and Mumbai", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN : 2350 – 0328), 9(12), 20210 – 20219. www.ijarset.com .

[25] Dhritikesh Chakrabarty (2022): "Method of Determination of Central Tendency of Non-negative Integral Valued Data: Application in Rainfall Data at Mumbai", Partners Universal International Research Journal (PUIRJ), ISSN: 2583-5602, 01(04), 67 – 74. www.puirj.com . DOI:10.5281/zenodo.7422267.

[26] Dhritikesh Chakrabarty (2023): "Determination of Tendency of Rainfall in India Described by Number of Rainy Days", Partners Universal International Research Journal (PUIRJ), ISSN: 2583-5602, 02(01), 95 – 102. www.puirj.com . DOI:10.5281/zenodo.7770100 .

[27] Dhritikesh Chakrabarty (2023): "Definition of Probability Based on Already Happened Outcomes: Application in Identifying Rainy and Non-Rainy Period", Partners Universal International Innovation Journal (PUIIJ), 01(04), 259 – 267. www.puiij.com . DOI:10.5281/zenodo.8282811. . https://www.researchgate.net/publication/373522632_Definition_of_Probability_Based_on_Already_Happened_Outcomes_Application_in_Identifying_Rainy_and_Non-Rainy_Period?

[28] Dhritikesh Chakrabarty (2023): "Determination of Tendency of Rainfall in India Described by Number of Rainy Days", Preprint, March 2023. DOI: 10.13140/RG.2.2.20709.99046 .

[29] Dhritikesh Chakrabarty (2023): "Numbers of Rainy Days at Chennai, Kolkata, Mumbai, and New Delhi: Most Likely to Occur", Partners Universal International Research Journal (PUIRJ), ISSN: 2583-5602, 02(03), 210 – 217. www.puirj.com . DOI:10.5281/zenodo.8372740. https://www.researchgate.net/publication/374265699_Numbers_of_Rainy_Days_at_Chennai_Kolkata_Mumbai_and_New_Delhi_Most_Likely_to_Occur?

[30] Dhritikesh Chakrabarty (2023): "Probability of Occurrence of Rainy Days: Non-Rainfall Tendency in India", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN: 2350 – 0328), 10(9), 21018 – 21025. www.ijarset.com . https://www.researchgate.net/publication/374338060_Probability_of_Occurrence_of_Rainy_Days_Non-Rainfall_Tendency_in_India?

[31] Fisher, R. A. (1922): "The Mathematical Foundations of Theoretical Statistics", Philosophical Transactions of the Royal Society of London, 222 (1922), 309 – 368. CrossRef Google Scholar .

[32] Ganchang He, Yaning Chen, Gonghuan Fang, Zhi (2022): "Li, Hydrometeorological Forecast of a Typical Watershed in an Arid Area Using Ensemble Kalman Filter", Water, 14(23), 3970. 10.3390/w14233970.

[33] Jongho Kim, Jaehyeon Lee, Dongkyun Kim, Boosik Kang (2019): "The role of rainfall spatial variability in estimating areal reduction factors", Journal of Hydrology, 568, 416 – 426. 10.1016/j.jhydrol.2018.11.014 .

[34] Kundu S., Khare D. & Mondal A. (2017): "Future changes in rainfall, temperature and reference evapotranspiration in the central India by least square support vector machine", Geoscience Frontiers, 8(3), 583 -596

[35] Kammun (2019), "Rainfall Analysis - A Review", International Research Journal of Engineering and Technology, 6(12) 2614 – 2617.
[36] Kolmogorov A. N. (1933): " Grunbegriffe der Wahrscheinlichkeits Rechnung ", Ergeb. Math. And ihrer Grensg., 2, 62 – 88. (The Monograph Published by Springer, Berlin,1933).
[37] Kolmogorov A. N. (1956): "Foundations of the Theory of Probability", 2nd English Edition, Chelsea Publishing Company, New York: A Translation of Grundbergriffe der Wahrscheinlichkeitsrechnung.
[38] Korbinian Breinl, Hannes Müller-Thomy & Günter Blöschl (2020): "Space–Time Characteristics of Areal Reduction Factors and Rainfall Processes", Journal of Hydrometeorology, 21(4), 671– 689. 10.1175/JHM-D-19-0228.1.
[39] Kassaw Beshaw Tessema, Alemseged Tamiru Haile et al (2020): "Negash Wagesho Amencho, Emad Habib, Effect of rainfall variability and gauge representativeness on satellite rainfall accuracy in a small upland watershed in southern Ethiopia", Hydrological Sciences Journal, 67(16), 2490 – 2504,. 10.1080/02626667.2020.1770766.
[40] Maistrov L. E. (1974): "Probability Theory: A Historical Sketch", Academic Press, New York & London.
[41] Namdev K., Madan S. (2021), "Study of Rainfall Variation in Parbhani District of Maharashtra (2000-2016)", International Journal of Creative Research Thoughts, 9(6), 135-140.
[42] Padhiary J., Das D.M., et al (2018), "Trend analysis of rainfall and temperature using the Mann kendall test in Jaraikela catchment of Brahmani river basin", International Journal of Agriculture Sciences, 10(19), 7309-7313.
[43] Peter C. Fishburn {1967}: "Preference-based Definitions of Subjective Probability", The Annals of Mathematical Statistics, 38(6) ,1605-1617.
[44] Rolando B. Chuaqui (1977): "A Semantical Definition of Probability", Studies in Logic and the Foundations of Mathematics, 89, 135-167.
[45] Søren Thorndahl, Jesper Ellerbæk Nielsen et al (2019): "Michael R. Rasmussen, Estimation of Storm-Centred Aerial Reduction Factors from Radar Rainfall for Design in Urban Hydrology", Water, 11(6), 1120. 10.3390/w11061120.
[46] Savage L. J. (1961). "The Foundation of Statistics Reconsidered: Proceedings of the Fourth Berkeley Symposium on mathematics and Probability.
[47] Tank G., Dongre P, et al (2021), "Rainfall Trend Analysis – A Review", International Research Journal of Engineering and Technology, 8(4), 4028-4030.
[48] Subodh Kant Pandey & H. L. Tiwari (2021): "RAINFALL TREND DETECTION – A REVIEW", International Journal of Creative Research Thoughts (IJCRT), 9(11), 521 – 524. www.ijcrt.org.
[49] von Mises R. (1931): "Wahrscheinlichkeits Rechnung. (English Edition: " Mathematical Theory of Probability and Statistics ", Academic Press, New York, 1964).
[50] Wing H, Gabriel B & Ashbindu S (2008): "Trends and Spatial distribution of annual and seasonal rainfall in Ethiopia", Int. J. Climatol, 28(13), 1723 –1734.